

# 10984 Double NP-hard

“Name and Class Year:  
Course to be Covered: (Course Number and Title)  
Reason for covering the course independently:”  
Hamilton College Application for Independent  
Coverage of Course Work

### Definitions

In this problem, a *graph* is a set of  $n$  vertices together with a set of  $m$  edges, where an *edge* is an unordered pair of different vertices (edges are undirected). The two vertices that comprise an edge are said to be that edge’s *endpoints*. A *vertex cover* of a given graph  $G$  is a subset  $C$  of its vertices, such that each edge of  $G$  has at least one of its endpoints in  $C$ . An *independent set* of a given graph  $G$  is a subset  $S$  of its vertices, such that no edge of  $G$  has both of its endpoints in  $S$ .

The problem of finding a *minimum vertex cover* (that is, a vertex cover of the smallest possible size) for any graph is NP-hard. The problem of finding a *maximum independent set* of any graph is also NP-hard. That is a formal way of saying that no one knows whether there exists an algorithm that runs in time polynomial in  $n$  and solves any one of the two problems.

We want to define a class of problems that are even harder than the NP-hard problems. We are going to call them “Double NP-hard”! Your job is to solve the first Double NP-hard problem.

### Problem

Given a graph  $G$ , find a subset  $C$  of its vertices that is both a *minimum vertex cover* and a *maximum independent set*.

### Input

The first line of input gives the number of cases,  $N$ .  $N$  test cases follow. Each one starts with two lines containing  $n$  ( $0 \leq n \leq 1000$ ) and  $m$  ( $0 \leq m \leq 100000$ ) as above. The next  $m$  lines will each describe an edge of  $G$  as a pair of different vertices, which are numbered from 1 to  $n$ .

### Output

For each test case, output one line containing ‘Case # $x$ :’ followed by either ‘Impossible’ if there is no answer or the size  $k$  of the set  $C$ . In the latter case, on the next line, print the  $k$  vertices of  $C$  in increasing order, separated by spaces. If there are multiple answers, print the lexicographically smallest one.

### Sample Input

```
4
2
1
1 2
0
0
10
0
4
```

4  
1 2  
2 3  
3 4  
4 1

### Sample Output

Case #1: 1  
1  
Case #2: 0

Case #3: Impossible  
Case #4: 2  
1 3