

# 10753 Exponential Function

In the course of Linear Algebra, the following theorem is proved:

**Theorem.** *Let  $A$  be a square matrix of size  $n$  with entries in  $\mathbf{C}$ . There are square matrices  $T$  and  $J$  of size  $n$  such that*

$$A = T^{-1}JT, \quad J = \begin{pmatrix} J_1 & \mathbb{0} & \mathbb{0} & \mathbb{0} \\ \mathbb{0} & J_2 & \mathbb{0} & \mathbb{0} \\ \mathbb{0} & \mathbb{0} & \ddots & \mathbb{0} \\ \mathbb{0} & \mathbb{0} & \mathbb{0} & J_k \end{pmatrix}$$

where  $J_i$  are Jordan cells:

$$J_i = \begin{pmatrix} \lambda_i & 1 & 0 & 0 & \dots & 0 \\ 0 & \lambda_i & 1 & 0 & \dots & 0 \\ \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ 0 & \dots & \dots & \dots & \lambda_i & 1 \\ 0 & \dots & \dots & \dots & \dots & \lambda_i \end{pmatrix}$$

Here  $\lambda_i$  is an eigenvalue of  $A$ .

The decomposition  $A = T^{-1}JT$ , where  $J$  is of the form described above, is called a *Jordan decomposition* of  $A$ . The Jordan decomposition of a matrix may fail to be unique.

Given a matrix  $A$ , we can define the matrix  $\exp A$  in the following way: if  $A = T^{-1}JT$  is a Jordan decomposition of  $A$ , then  $\exp A = T^{-1}J'T$

$$J' = \begin{pmatrix} J'_1 & \mathbb{0} & \mathbb{0} \\ \mathbb{0} & \ddots & \mathbb{0} \\ \mathbb{0} & \mathbb{0} & J'_k \end{pmatrix}, \quad J'_i = \begin{pmatrix} \frac{e^{\lambda_i}}{0!} & \dots & \dots & \frac{e^{\lambda_i}}{m_i!} \\ 0 & \frac{e^{\lambda_i}}{0!} & \dots & \frac{e^{\lambda_i}}{(m_i-1)!} \\ \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & \frac{e^{\lambda_i}}{0!} \end{pmatrix}$$

Here  $m_i$  is the size of  $J_i$ . If  $k \leq l$ , then the number in the  $k$ -th row and  $l$ -th column of  $J'_i$  is

$$j_{kl} = \frac{e^{\lambda_i}}{(l-k)!},$$

otherwise it is 0.

It can be proved that  $\exp A$  is independent of the Jordan decomposition of  $A$  used. It can also be proved that if  $A$  is real-valued, then  $\exp A$  is also real-valued. Your task is: given a matrix  $A$ , compute  $\exp A$ .

For example, if

$$A = \begin{pmatrix} 3 & 0 \\ 1 & 3 \end{pmatrix},$$

then

$$J = \begin{pmatrix} 3 & 1 \\ 0 & 3 \end{pmatrix}, T = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, J' = \begin{pmatrix} e^3 & e^3 \\ 0 & e^3 \end{pmatrix}$$

and

$$\exp A = \begin{pmatrix} e^3 & 0 \\ e^3 & e^3 \end{pmatrix} \approx \begin{pmatrix} 20.086 & 0 \\ 20086 & 20086 \end{pmatrix}$$

### Input

The first line of the input contains the number of the test cases, which is at most 15. The descriptions of the test cases follow. The first line of a test case description contains one integer  $N$  ( $1 \leq N \leq 8$ ), denoting the size of the matrix  $A$ . Each of the next  $N$  lines contains  $N$  integers separated by spaces, describing the matrix  $A$ . It is guaranteed that the entries of  $A$  are between 0 and 5. The test cases are separated by blank lines.

### Output

For each test case in the input, output  $N$  lines, each containing  $N$  integers separated by spaces, describing the matrix  $\exp A$ . The numbers must have at least three digits after the decimal point. Print a blank line between test cases.

### Sample Input

```
2

2
3 0
1 3

1 5
0 1
```

### Sample Output

```
20.086 0.000
20.086 20.086

2.718 13.591
0.000 2.718
```