

## 10444 Multi-Peg Towers of Hanoi

In 3-Peg Tower of Hanoi problem  $n$  disks, all of unequal radiuses are placed initially on a peg 1 with smaller disks on top of larger disks. Using empty peg 2 the whole tower should be shifted to peg 3 with the condition that only one disk at a time from top of any peg can be moved to the top of another peg. Never a larger disk can be placed on top of a smaller one. While shifting the tower in minimum moves requires a simple recursive strategy of shifting  $n - 1$  disks to the only intermediate peg, number of moves required is exponential. That is if there are  $n$  disks to move then  $2^n - 1$  moves are necessary. If there are  $p > 3$  pegs then the task becomes much easier but now finding the optimal strategy is more difficult. In fact nobody has ever been able to prove that a certain strategy is optimal. Researchers, however, have found a strategy that many believe to be optimal, although could not prove so. This strategy is known to be *Presumed Optimal Solution* (POS). In POS the strategy is the following.

In shifting a tower of  $n$  disks in  $p$ -peg system a certain optimal number  $n_1$  disks are shifted to an intermediate peg  $P_i$ . Then remaining  $n - n_1$  disks are shifted to destination using a total of  $p - 1$  pegs, since peg  $P_i$  is loaded with smaller disks, and therefore, cannot be used for transfer of larger disks. This strategy is employed recursively for solving any sub-problem. While such a strategy can result in different POS solutions the number of moves remains the same.



Fig: A four-peg tower of Hanoi

It is known that in POS solutions each disk requires  $2^k$  moves to reach destination for some integer  $k$ . Furthermore, maximum number of disks each of which requires  $2^k$  moves to reach destination is equal to  ${}^{p-3+k}C_k$  (number of combinations of  $(p - 3 + k)$  things taken  $k$  at a time), provided that such a number of disks is available. Moreover, in a POS strategy it is possible to transfer disks greedily. That is, if there are disks each of which requires  $2^{k+1}$  moves to reach destination, then number of disks each of which require  $2^k$  moves to reach destination is  ${}^{p-3+k}C_k$  (number of combinations of  $(p - 3 + k)$  things taken  $k$  at a time). Given  $n, p$  you will have to determine the minimum number of moves required to move the  $n$  disks from source to destination using  $p$  pegs and the process described above.

### Input

The input file contains several lines of input. Each line contains three integers  $n$  ( $0 \leq n \leq 200$ ),  $p$  ( $3 < p \leq 20$ ). The meanings of these two integers are explained in the problem statement. Input is terminated by a line containing two zeroes. This line should not be processed.

## Output

For each line of input (that are asked to be processed) produce one line of output. This line should contain the serial of the output followed by an integer  $N$  that indicates the number of movements required to move all the disks from source peg to destination peg.

## Sample Input

```
3 4
4 4
10 4
10 5
0 0
```

## Sample Output

```
Case 1: 5
Case 2: 9
Case 3: 49
Case 4: 31
```